Lecture -2

Vector Representation & Coordinate Systems

VECTOR REPRESENTATION

3 PRIMARY COORDINATE SYSTEMS:

- RECTANGULAR
- CYLINDRICAL
- SPHERICAL

Choice is based on symmetry of problem

Examples:

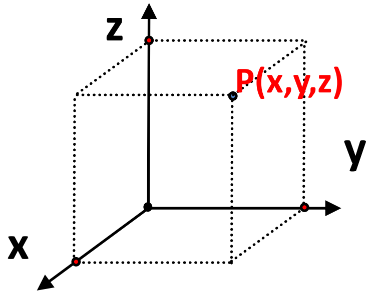
Sheets - RECTANGULAR

Wires/Cables - CYLINDRICAL

Spheres - SPHERICAL

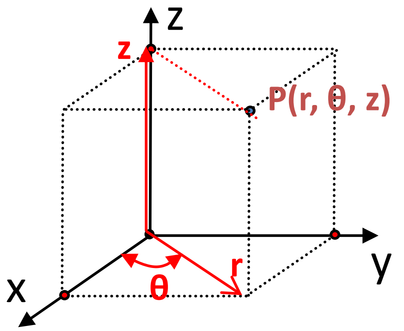
Orthogonal Coordinate Systems: (coordinates mutually perpendicular)

Cartesian Coordinates



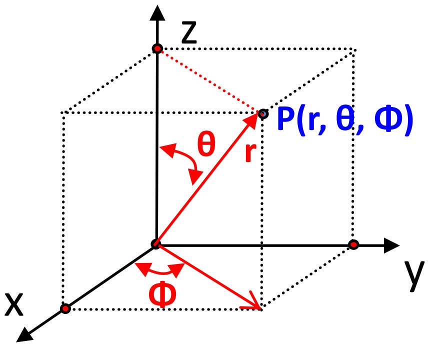
Rectangular Coordinates

P (x,y,z)



Cylindrical Coordinates

P (r, Θ, z)

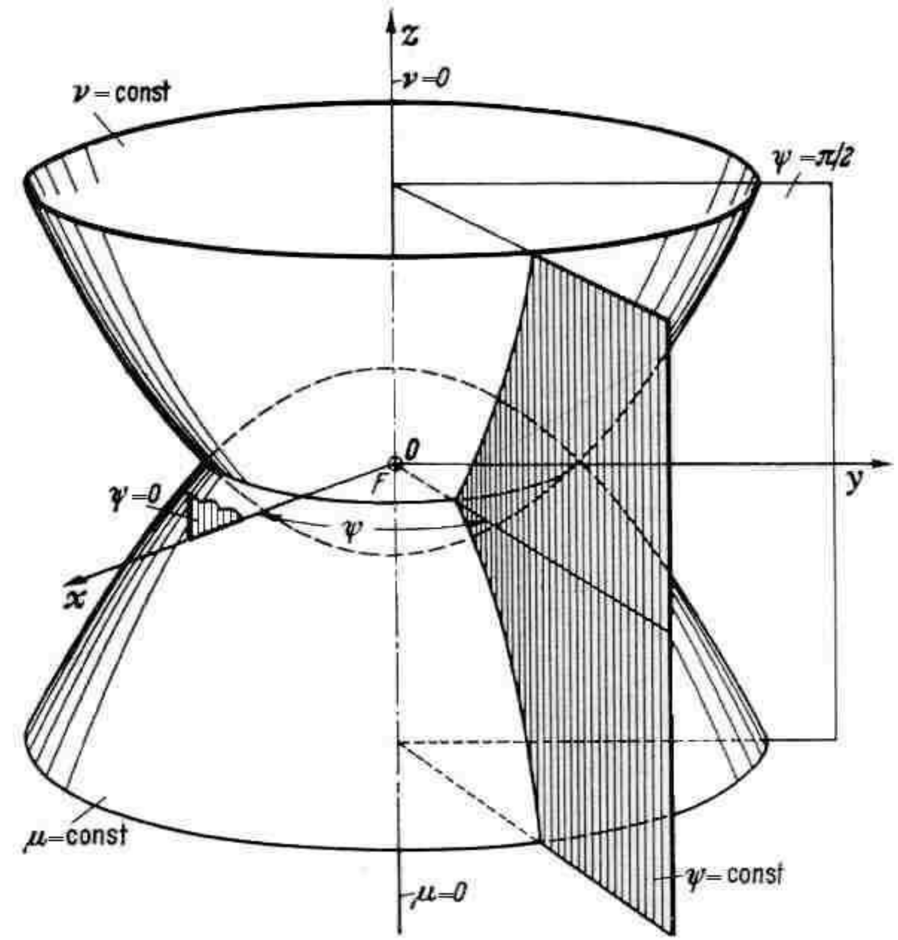


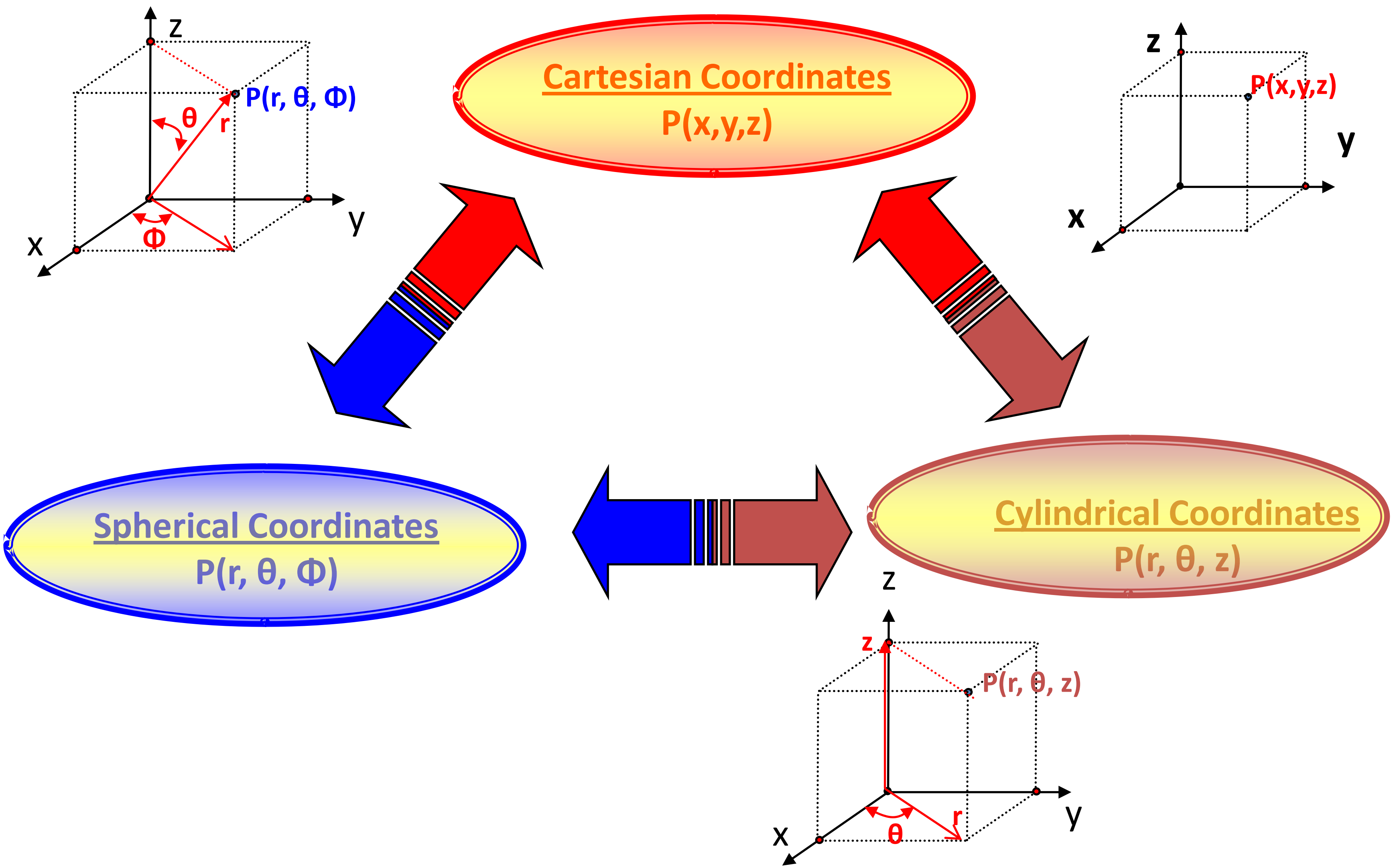
Spherical Coordinates

P (r, Θ, Φ)

-**Parabolic Cylindrical Coordinates (u,v,z)**
-**Paraboloidal Coordinates (u, v, Φ)**
-**Elliptic Cylindrical Coordinates (u, v, z)**
-**Prolate Spheroidal Coordinates (ξ, η, ϕ)**
-**Oblate Spheroidal Coordinates (ξ, η, ϕ)**
-**Bipolar Coordinates (u,v,z)**
-**Toroidal Coordinates (u, v, Φ)**
-**Conical Coordinates (λ, μ, ν)**
-**Confocal Ellipsoidal Coordinate (λ, μ, ν)**
-**Confocal Paraboloidal Coordinate (λ, μ, ν)**

Parabolic Cylindrical Coordinates





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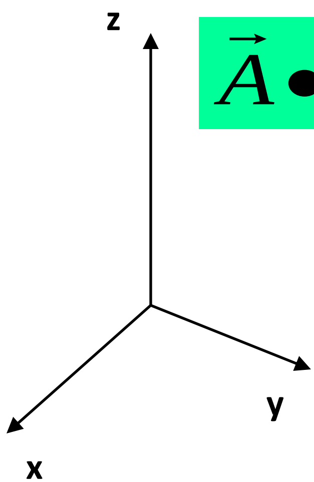
VECTOR NOTATION

VECTOR NOTATION:

$$\vec{A} = A_x\hat{a}_x + A_y\hat{a}_y + A_z\hat{a}_z$$

Rectangular or Cartesian Coordinate System



$$\vec{A} \bullet \vec{B} = A_xB_x + A_yB_y + A_zB_z$$

Dot Product
(SCALAR)

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

Cross Product
(VECTOR)

$$\left|\vec{A}\right| = \left(A_x^2 + A_y^2 + A_z^2\right)^{\frac{1}{2}}$$

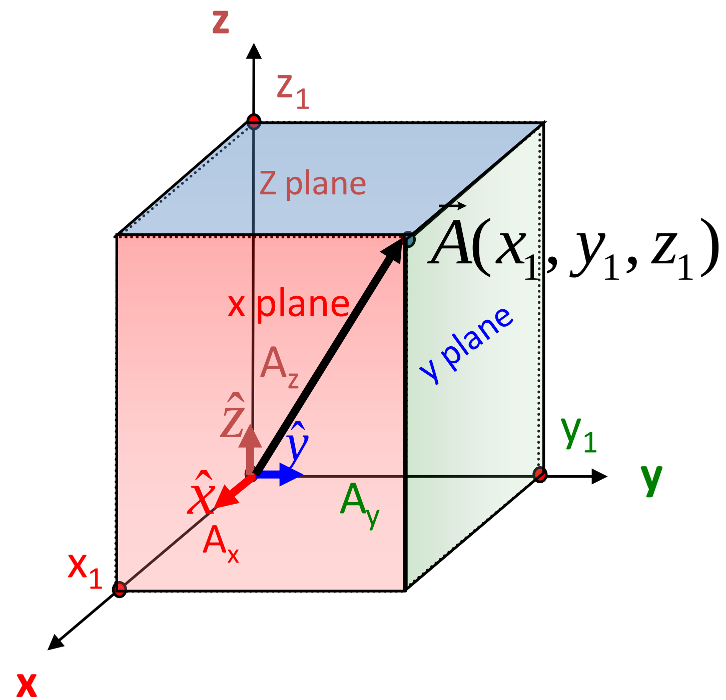
Magnitude of vector

Cartesian Coordinates

(x, y, z)

Vector representation

$$\vec{A} = \hat{x}A_x + \hat{y}A_y + \hat{z}A_z$$



Magnitude of A

$$\left|\vec{A}\right| = \sqrt[+]{\vec{A}\cdot\vec{A}} = \sqrt[+]{A_x^2 + A_y^2 + A_z^2}$$

Position vector A

$$\hat{x}x_1 + \hat{y}y_1 + \hat{z}z_1$$

Base vector properties

$$\hat{x}\cdot\hat{x} = \hat{y}\cdot\hat{y} = \hat{z}\cdot\hat{z} = 1$$

$$\hat{x}\cdot\hat{y} = \hat{y}\cdot\hat{z} = \hat{z}\cdot\hat{x} = 0$$

$$\hat{x}\times\hat{y} = \hat{z}$$

$$\hat{y}\times\hat{z} = \hat{x}$$

$$\hat{z}\times\hat{x} = \hat{y}$$

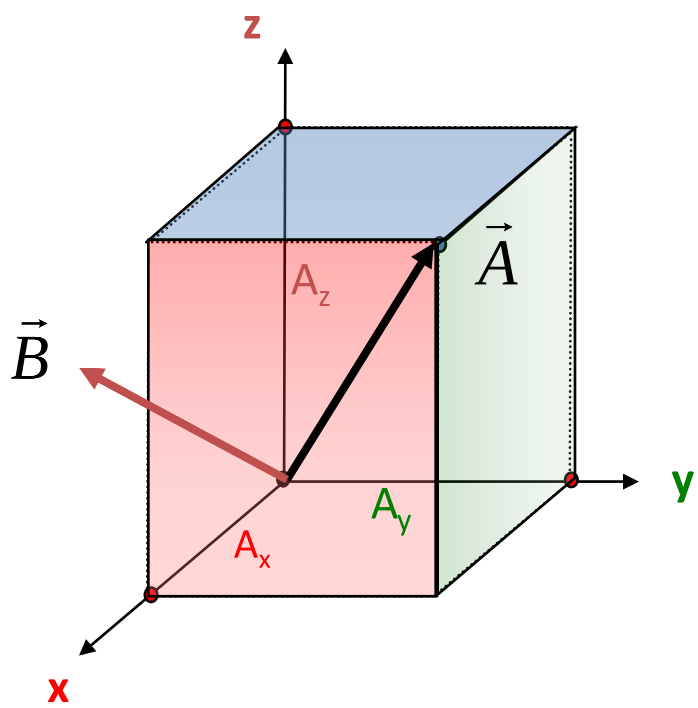
Cartesian Coordinates

Dot product:

$$\vec{A}\cdot\vec{B} = A_x B_x + A_y B_y + A_z B_z$$

Cross product:

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$



VECTOR REPRESENTATION: CYLINDRICAL COORDINATES

UNIT VECTORS:

$$\left(\hat{a}_r \quad \hat{a}_\phi \quad \hat{a}_z\right)$$

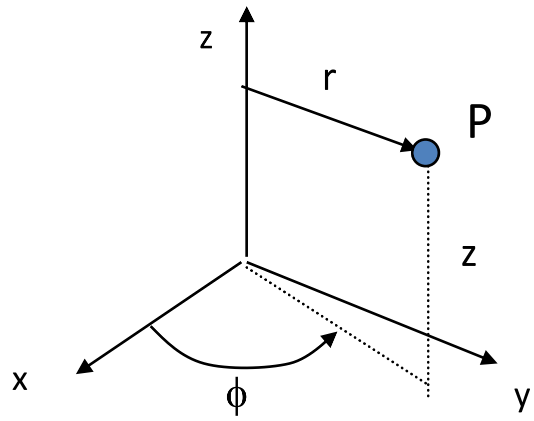
Cylindrical representation uses: r, ϕ, z

$$\vec{A} = A_r\hat{a}_r + A_\phi\hat{a}_\phi + A_z\hat{a}_z$$

$$\vec{A} \bullet \vec{B} = A_rB_r + A_\phi B_\phi + A_zB_z$$

Dot Product

(SCALAR)



VECTOR REPRESENTATION: SPHERICAL COORDINATES

UNIT VECTORS:

$$\left(\hat{a}_r \quad \hat{a}_\theta \quad \hat{a}_\phi\right)$$

Spherical representation uses: r, θ, ϕ

$$\vec{A} = A_r\hat{a}_r + A_\theta\hat{a}_\theta + A_\phi\hat{a}_\phi$$

$$\vec{A} \bullet \vec{B} = A_r B_r + A_\theta B_\theta + A_\phi B_\phi$$

Dot Product

(SCALAR)